Національний технічний університет України

«Київський політехнічний інститут імені Ігоря Сікорського»

Факультет інформатики та обчислювальної техніки

Кафедра обчислювальної техніки

**Методи оптимізації та планування експерименту**

**Лабораторна робота №6**

«Проведення трьохфакторного експерименту

при використанні рівняння регресії з квадратичними членами»

Виконав:

студент групи ІВ-83

Грисюк М. О.

Залікова книжка №8306

Перевірив Регіда П. Г.

Київ - 2020 р.



**from** copy **import** deepcopy  
**from** datetime **import** datetime  
**from** math **import** sqrt  
**from** random **import** random  
**import** numpy **as** np  
**from** prettytable **import** PrettyTable  
  
x1\_min = -5  
x1\_max = 15  
x2\_min = 25  
x2\_max = 45  
x3\_min = 15  
x3\_max = 45  
  
koefs = [6.8, 1.7, 5.7, 9.9, 8.9, 0.5, 6.1, 9.9, 0.7, 3.2, 8.4]  
x\_average\_max = (x1\_max + x2\_max + x3\_max) / 3  
x\_average\_min = (x1\_min + x2\_min + x3\_min) / 3  
y\_max = 200 + x\_average\_max  
y\_min = 200 + x\_average\_min  
  
print(**"-"** \* 100)  
print(**'Значення за варіантом:'**)  
print(**'x1\_min = '**,x1\_min)  
print(**'x1\_max = '**,x1\_max)  
print(**'x2\_min = '**,x2\_min)  
print(**'x2\_max = '**,x2\_max)  
print(**'x3\_min = '**,x3\_min)  
print(**'x3\_max = '**,x3\_max)  
print(**'y\_min = '**,y\_min)  
print(**'y\_max = '**,y\_max)  
print(**'f(x1,x2,x3) = 6,8+1,7\*x1+5,7\*x2+9,9\*x3+8,9\*x1\*x1+0,5\*x2\*x2+6,1\*x3\*x3+9,9\*x1\*x2+0,7\*x1\*x3+3,2\*x2\*x3+8,4\*x1\*x2\*x3'**)  
print(**"-"** \* 100)  
  
**def** replace\_column(list\_: list, column, list\_replace):  
 list\_ = deepcopy(list\_)  
 **for** i **in** range(len(list\_)):  
 list\_[i][column] = list\_replace[i]  
 **return** list\_  
  
  
**def** append\_to\_list\_x(x: list, variant: int):  
 **if** variant == 1:  
 **for** i **in** range(len(x)):  
 x[i].append(x[i][1] \* x[i][2])  
 x[i].append(x[i][1] \* x[i][3])  
 x[i].append(x[i][2] \* x[i][3])  
 x[i].append(x[i][1] \* x[i][2] \* x[i][3])  
 **if** variant == 2:  
 **for** i **in** range(len(x)):  
 x[i].append(x[i][1] \* x[i][2])  
 x[i].append(x[i][1] \* x[i][3])  
 x[i].append(x[i][2] \* x[i][3])  
 x[i].append(x[i][1] \* x[i][2] \* x[i][3])  
 x[i].append(x[i][1] \* x[i][1])  
 x[i].append(x[i][2] \* x[i][2])  
 x[i].append(x[i][3] \* x[i][3])  
 **for** i **in** range(len(x)):  
 **for** j **in** range(len(x[i])):  
 **if** round(x[i][j], 3) == 0:  
 x[i][j] = 0  
 x[i][j] = round(x[i][j], 3)  
  
  
**def** get\_value(table: dict, key: int):  
 value = table.get(key)  
 **if** value **is not None**:  
 **return** value  
 **for** i **in** table:  
 **if** type(i) == range **and** key **in** i:  
 **return** table.get(i)  
  
  
**def** main(m, n):  
 **if** n == 14:  
 const\_l = 1.73  
 print(  
 **'ŷ = b0 + b1 \* x1 + b2 \* x2 + b3 \* x3 + b12 \* x1 \* x2 + b13 \* x1 \* x3 + b23 \* x2 \* x3 + b123 \* x1 \* x2 \* '  
 'x3 + b11 \* x1 \* x1 + b22 \* x2 \* x2 + b33 \* x3 \* x3'**)  
 norm\_x = [  
 [+1, -1, -1, -1],  
 [+1, -1, +1, +1],  
 [+1, +1, -1, +1],  
 [+1, +1, +1, -1],  
 [+1, -1, -1, +1],  
 [+1, -1, +1, -1],  
 [+1, +1, -1, -1],  
 [+1, +1, +1, +1],  
 [+1, -const\_l, 0, 0],  
 [+1, const\_l, 0, 0],  
 [+1, 0, -const\_l, 0],  
 [+1, 0, const\_l, 0],  
 [+1, 0, 0, -const\_l],  
 [+1, 0, 0, const\_l],  
 ]  
  
 delta\_x1 = (x1\_max - x1\_min) / 2  
 delta\_x2 = (x2\_max - x2\_min) / 2  
 delta\_x3 = (x2\_max - x3\_min) / 2  
 x01 = (x1\_min + x1\_max) / 2  
 x02 = (x2\_min + x2\_max) / 2  
 x03 = (x3\_min + x3\_max) / 2  
  
 x = [  
 [1, x1\_min, x2\_min, x3\_min],  
 [1, x1\_min, x2\_max, x3\_max],  
 [1, x1\_max, x2\_min, x3\_max],  
 [1, x1\_max, x2\_max, x3\_min],  
 [1, x1\_min, x2\_min, x3\_max],  
 [1, x1\_min, x2\_max, x3\_min],  
 [1, x1\_max, x2\_min, x3\_min],  
 [1, x1\_max, x2\_max, x3\_max],  
 [1, -const\_l \* delta\_x1 + x01, x02, x03],  
 [1, const\_l \* delta\_x1 + x01, x02, x03],  
 [1, x01, -const\_l \* delta\_x2 + x02, x03],  
 [1, x01, const\_l \* delta\_x2 + x02, x03],  
 [1, x01, x02, -const\_l \* delta\_x3 + x03],  
 [1, x01, x02, const\_l \* delta\_x3 + x03],  
 ]  
  
 append\_to\_list\_x(norm\_x, variant=2)  
 append\_to\_list\_x(x, variant=2)  
  
 **if** n == 8:  
 print(  
 **'ŷ = b0 + b1 \* x1 + b2 \* x2 + b3 \* x3 + b12 \* x1 \* x2 + b13 \* x1 \* x3 + b23 \* x2 \* x3 + b123 \* x1 \* x2 \* x3'** )  
 norm\_x = [  
 [+1, -1, -1, -1],  
 [+1, -1, +1, +1],  
 [+1, +1, -1, +1],  
 [+1, +1, +1, -1],  
 [+1, -1, -1, +1],  
 [+1, -1, +1, -1],  
 [+1, +1, -1, -1],  
 [+1, +1, +1, +1]  
 ]  
  
 x = [  
 [1, x1\_min, x2\_min, x3\_min],  
 [1, x1\_min, x2\_max, x3\_max],  
 [1, x1\_max, x2\_min, x3\_max],  
 [1, x1\_max, x2\_max, x3\_min],  
 [1, x1\_min, x2\_min, x3\_max],  
 [1, x1\_min, x2\_max, x3\_min],  
 [1, x1\_max, x2\_min, x3\_min],  
 [1, x1\_max, x2\_max, x3\_max]  
 ]  
  
 append\_to\_list\_x(norm\_x, variant=1)  
 append\_to\_list\_x(x, variant=1)  
  
 **if** n == 4:  
 print(**'ŷ = b0 + b1 \* x1 + b2 \* x2 + b3 \* x3'**)  
 norm\_x = [  
 [+1, -1, -1, -1],  
 [+1, -1, +1, +1],  
 [+1, +1, -1, +1],  
 [+1, +1, +1, -1],  
 ]  
 x = [  
 [1, x1\_min, x2\_min, x3\_min],  
 [1, x1\_min, x2\_max, x3\_max],  
 [1, x1\_max, x2\_min, x3\_max],  
 [1, x1\_max, x2\_max, x3\_min],  
 ]  
 **if** n == 14:  
 y = [[round(sum([koefs[j] \* i[j] **for** j **in** range(len(koefs))]) + random() \* 10 - 5, 3) **for** k **in** range(m)] **for** i  
 **in** x]  
 **else**:  
 y = np.random.randint(y\_min, y\_max, size=(n, m))  
 *# y = np.random.randint(y\_min, y\_max, size=(n, m))* y\_av = list(np.average(y, axis=1))  
  
 **for** i **in** range(len(y\_av)):  
 y\_av[i] = round(y\_av[i], 3)  
  
 **if** n == 14:  
 t1 = PrettyTable([**'N'**, **'norm\_x\_0'**, **'norm\_x\_1'**, **'norm\_x\_2'**, **'norm\_x\_3'**, **'norm\_x\_1\_x\_2'**, **'norm\_x\_1\_x\_3'**,  
 **'norm\_x\_2\_x\_3'**, **'norm\_x\_1\_x\_2\_x\_3'**, **'norm\_x\_1\_x\_1'**, **'norm\_x\_2\_x\_2'**, **'norm\_x\_3\_x\_3'**, ] +  
 [**f'y\_{**i + 1**}' for** i **in** range(m)] + [**'y\_av'**])  
 t2 = PrettyTable([**'N'**, **'x\_0'**, **'x\_1'**, **'x\_2'**, **'x\_3'**, **'x\_1\_x\_2'**, **'x\_1\_x\_3'**, **'x\_2\_x\_3'**, **'x\_1\_x\_2\_x\_3'**, **'x\_1\_x\_1'**,  
 **'x\_2\_x\_2'**, **'x\_3\_x\_3'**] + [**f'y\_{**i + 1**}' for** i **in** range(m)] + [**'y\_av'**])  
  
 **if** n == 8:  
 t = PrettyTable([**'N'**, **'norm\_x\_0'**, **'norm\_x\_1'**, **'norm\_x\_2'**, **'norm\_x\_3'**, **'norm\_x\_1\_x\_2'**, **'norm\_x\_1\_x\_3'**,  
 **'norm\_x\_2\_x\_3'**, **'norm\_x\_1\_x\_2\_x\_3'**, **'x\_0'**, **'x\_1'**, **'x\_2'**, **'x\_3'**, **'x\_1\_x\_2'**, **'x\_1\_x\_3'**,  
 **'x\_2\_x\_3'**, **'x\_1\_x\_2\_x\_3'**] + [**f'y\_{**i + 1**}' for** i **in** range(m)] + [**'y\_av'**])  
 **if** n == 4:  
 t = PrettyTable(  
 [**'N'**, **'norm\_x\_0'**, **'norm\_x\_1'**, **'norm\_x\_2'**, **'norm\_x\_3'**, **'x\_0'**, **'x\_1'**, **'x\_2'**, **'x\_3'**] +  
 [**f'y\_{**i + 1**}' for** i **in** range(m)] + [**'y\_av'**])  
  
 **if** n == 14:  
 **for** i **in** range(n):  
 t1.add\_row([i + 1] + list(norm\_x[i]) + list(y[i]) + [y\_av[i]])  
 t2.add\_row([i + 1] + list(x[i]) + list(y[i]) + [y\_av[i]])  
 print(t1)  
 print(t2)  
 **else**:  
 **for** i **in** range(n):  
 t.add\_row([i + 1] + list(norm\_x[i]) + list(x[i]) + list(y[i]) + [y\_av[i]])  
 print(t)  
  
 m\_ij = []  
 **for** i **in** range(len(x[0])):  
 m\_ij.append([round(sum([x[k][i] \* x[k][j] **for** k **in** range(len(x))]) / 14, 3) **for** j **in** range(len(x[i]))])  
  
 k\_i = []  
 **for** i **in** range(len(x[0])):  
 a = sum(y\_av[j] \* x[j][i] **for** j **in** range(len(x))) / 14  
 k\_i.append(a)  
  
 det = np.linalg.det(m\_ij)  
 det\_i = [np.linalg.det(replace\_column(m\_ij, i, k\_i)) **for** i **in** range(len(k\_i))]  
  
 b\_i = [round(i / det, 3) **for** i **in** det\_i]  
 **if** n == 14:  
 print(  
 **f"\nНормалізоване рівняння регресії: "  
 f"y = {**b\_i[0]**:.5f} + {**b\_i[1]**:.5f} \* x1 + {**b\_i[2]**:.5f} \* x2 + "  
 f"{**b\_i[3]**:.5f} \* x3 + {**b\_i[4]**:.5f} \* x1 \* x2 + "  
 f"{**b\_i[5]**:.5f} \* x1 \* x3 + {**b\_i[6]**:.5f} \* x2 \* x3 + {**b\_i[7]**:.5f} \* x1 \* x2 \* x3 + {**b\_i[8]**:.5f} \* x1 \* x1 + "  
 f"{**b\_i[9]**:.5f} \* x2 \* x2 + {**b\_i[10]**:.5f} \* x3 \* x3"**)  
 **if** n == 8:  
 print(  
 **f"\nНормалізоване рівняння регресії: "  
 f"y = {**b\_i[0]**:.5f} + {**b\_i[1]**:.5f} \* x1 + {**b\_i[2]**:.5f} \* x2 + "  
 f"{**b\_i[3]**:.5f} \* x3 + {**b\_i[4]**:.5f} \* x1 \* x2 + "  
 f"{**b\_i[5]**:.5f} \* x1 \* x3 + {**b\_i[6]**:.5f} \* x2 \* x3 + {**b\_i[7]**:.5f} \* x1 \* x2 \* x3"**)  
 **if** n == 4:  
 print(  
 **f"\nНормалізоване рівняння регресії: "  
 f"y = {**b\_i[0]**:.5f} + {**b\_i[1]**:.5f} \* x1 + {**b\_i[2]**:.5f} \* x2 + {**b\_i[3]**:.5f} \* x3\n"**)  
  
 check\_i = [round(sum(b\_i[j] \* i[j] **for** j **in** range(len(b\_i))), 3) **for** i **in** x]  
 **for** i **in** range(len(check\_i)):  
 print(**f'ŷ{**i + 1**} = {**check\_i[i]**}, y\_av{**i + 1**} = {**y\_av[i]**}'**)  
  
 print(**"-"** \* 100)  
 print(**"Критерій Кохрена"**)  
 start\_time = datetime.now()  
 f\_1 = m - 1  
 f\_2 = n  
 s\_i = [sum([(i - y\_av[j]) \*\* 2 **for** i **in** y[j]]) / m **for** j **in** range(len(y))]  
 g\_p = max(s\_i) / sum(s\_i)  
  
 table = {2: 0.75, 3: 0.6841, 4: 0.6287, 5: 0.5892, 6: 0.5598, 7: 0.5365, 8: 0.5175, 9: 0.5017, 10: 0.4884,  
 range(11, 17): 0.4366, range(17, 37): 0.3720, range(37, 2 \*\* 100): 0.3093}  
 g\_t = get\_value(table, m)  
  
 **if** g\_p < g\_t:  
 print(**f"Тест тривав {**(datetime.now() - start\_time).total\_seconds()**} сек."**)  
 print(**f"Дисперсія однорідна: Gp = {**g\_p**:.5} < Gt = {**g\_t**}"**)  
 **else**:  
 print(**f"Тест тривав {**(datetime.now() - start\_time).total\_seconds()**} сек."**)  
 print(**f"Дисперсія неоднорідна: Gp = {**g\_p**:.5} > Gt = {**g\_t**}\nПочнемо знову з m = m + 1 = {**m + 1**}"**)  
 **return** main(m=m + 1, n=n)  
  
 print(**"-"** \* 100)  
 print(**"Критерій Стьюдента"**)  
 start\_time = datetime.now()  
 s2\_b = sum(s\_i) / n  
 s2\_beta\_s = s2\_b / (n \* m)  
 s\_beta\_s = sqrt(s2\_beta\_s)  
 beta\_i = [sum([norm\_x[i][j] \* y\_av[i] **for** i **in** range(len(norm\_x))]) / n **for** j **in** range(len(norm\_x[0]))]  
 beta\_i = [round(i, 3) **for** i **in** beta\_i]  
  
 t = [abs(i) / s\_beta\_s **for** i **in** beta\_i]  
 **if** n == 14:  
 beta\_i = b\_i  
 f\_3 = f\_1 \* f\_2  
 t\_table = {4: 2.776, 5: 2.571, 6: 2.447, 7: 2.365, 8: 2.306, 9: 2.262, 10: 2.228, 11: 2.201, 12: 2.179, 13: 2.160,  
 14: 2.145, 15: 2.131, 16: 2.120, 17: 2.110, 18: 2.101, 19: 2.093, 20: 2.086, 21: 2.08, 22: 2.074,  
 23: 2.069, 24: 2.064, range(25, 30): 2.06, range(30, 40): 2.042, range(40, 60): 2.021, range(60, 100): 2,  
 range(100, 2 \*\* 100): 1.96}  
 d = deepcopy(len(beta\_i))  
 **for** i **in** range(len(t)):  
 **if** get\_value(t\_table, f\_3) > t[i]:  
 beta\_i[i] = 0  
 d -= 1  
 print(**f"Тест тривав {**(datetime.now() - start\_time).total\_seconds()**} сек."**)  
 print(**"-"** \* 100)  
 **if** n == d:  
 n = 8 **if** n == 4 **else** 14  
 print(**f"n=d\nПочнемо знову з n = {**n**} and m = {**m**}"**)  
 **return** main(m=m, n=n)  
 **if** n == 14:  
 print(  
 **f"Нормалізоване спрощене рівяння регресії: "  
 f"y = {**beta\_i[0]**:.5f} + {**beta\_i[1]**:.5f} \* x1 + "  
 f"{**beta\_i[2]**:.5f} \* x2 + {**beta\_i[3]**:.5f} \* x3 + {**beta\_i[4]**:.5f} \* x1 \* x2 + "  
 f"{**beta\_i[5]**:.5f} \* x1 \* x3 + {**beta\_i[6]**:.5f} \* x2 \* x3 + {**beta\_i[7]**:.5f} \* x1 \* x2 \* x3 + "  
 f"{**beta\_i[8]**:.5f} \* x1 \* x1 + {**beta\_i[9]**:.5f} \* x2 \* x2 + {**beta\_i[10]**:.5f} \* x3 \* x3"**)  
 check\_i = [round(sum(beta\_i[j] \* i[j] **for** j **in** range(len(beta\_i))), 3) **for** i **in** x]  
  
 **if** n == 8:  
 print(  
 **f"Нормалізоване рівняння регресії: "  
 f"y = {**beta\_i[0]**:.5f} + {**beta\_i[1]**:.5f} \* x1 + {**beta\_i[2]**:.5f} \* x2 + "  
 f"{**beta\_i[3]**:.5f} \* x3 + {**beta\_i[4]**:.5f} \* x1 \* x2 + "  
 f"{**beta\_i[5]**:.5f} \* x1 \* x3 + {**beta\_i[6]**:.5f} \* x2 \* x3 + {**beta\_i[7]**:.5f} \* x1 \* x2 \* x3"**)  
 check\_i = [round(sum(beta\_i[j] \* i[j] **for** j **in** range(len(beta\_i))), 3) **for** i **in** norm\_x]  
  
 **if** n == 4:  
 print(  
 **f"Нормалізоване рівняння регресії: "  
 f"y = {**beta\_i[0]**:.5f} + {**beta\_i[1]**:.5f} \* x1 + {**beta\_i[2]**:.5f} \* x2 + "  
 f"{**beta\_i[3]**:.5f} \* x3"**)  
 check\_i = [round(sum(beta\_i[j] \* i[j] **for** j **in** range(len(beta\_i))), 3) **for** i **in** norm\_x]  
  
 **for** i **in** range(len(check\_i)):  
 print(**f'ŷ{**i + 1**} = {**check\_i[i]**}, y\_av{**i + 1**} = {**y\_av[i]**}'**)  
  
 print(**"-"** \* 100)  
 print(**"Критерій Фішера"**)  
 start\_time = datetime.now()  
 f\_4 = n - d  
 s2\_ad = m / f\_4 \* sum([(check\_i[i] - y\_av[i]) \*\* 2 **for** i **in** range(len(y\_av))])  
 f\_p = s2\_ad / s2\_b  
 f\_t = {  
 1: [164.4, 199.5, 215.7, 224.6, 230.2, 234, 235.8, 237.6],  
 2: [18.5, 19.2, 19.2, 19.3, 19.3, 19.3, 19.4, 19.4],  
 3: [10.1, 9.6, 9.3, 9.1, 9, 8.9, 8.8, 8.8],  
 4: [7.7, 6.9, 6.6, 6.4, 6.3, 6.2, 6.1, 6.1],  
 5: [6.6, 5.8, 5.4, 5.2, 5.1, 5, 4.9, 4.9],  
 6: [6, 5.1, 4.8, 4.5, 4.4, 4.3, 4.2, 4.2],  
 7: [5.5, 4.7, 4.4, 4.1, 4, 3.9, 3.8, 3.8],  
 8: [5.3, 4.5, 4.1, 3.8, 3.7, 3.6, 3.5, 3.5],  
 9: [5.1, 4.3, 3.9, 3.6, 3.5, 3.4, 3.3, 3.3],  
 10: [5, 4.1, 3.7, 3.5, 3.3, 3.2, 3.1, 3.1],  
 11: [4.8, 4, 3.6, 3.4, 3.2, 3.1, 3, 3],  
 12: [4.8, 3.9, 3.5, 3.3, 3.1, 3, 2.9, 2.9],  
 13: [4.7, 3.8, 3.4, 3.2, 3, 2.9, 2.8, 2.8],  
 14: [4.6, 3.7, 3.3, 3.1, 3, 2.9, 2.8, 2.7],  
 15: [4.5, 3.7, 3.3, 3.1, 2.9, 2.8, 2.7, 2.7, 2.7, 2.7, 2.6, 2.6],  
 16: [4.5, 3.6, 3.2, 3, 2.9, 2.7, 2.6, 2.6],  
 17: [4.5, 3.6, 3.2, 3, 2.8, 2.7, 2.5, 2.3],  
 18: [4.4, 3.6, 3.2, 2.9, 2.8, 2.7, 2.5, 2.3],  
 19: [4.4, 3.5, 3.1, 2.9, 2.7, 2.7, 2.4, 2.3],  
 range(20, 22): [4.4, 3.5, 3.1, 2.8, 2.7, 2.7, 2.4, 2.3],  
 range(22, 24): [4.3, 3.4, 3.1, 2.8, 2.7, 2.6, 2.4, 2.3],  
 range(24, 26): [4.3, 3.4, 3, 2.8, 2.6, 2.5, 2.3, 2.2],  
 range(26, 28): [4.2, 3.4, 3, 2.7, 2.6, 2.5, 2.3, 2.2],  
 range(28, 30): [4.2, 3.3, 3, 2.7, 2.6, 2.4, 2.3, 2.1],  
 range(30, 40): [4.2, 3.3, 3, 2.7, 2.6, 2.4, 2.3, 2.1, 2, 2, 2, 2],  
 range(40, 60): [4.1, 3.2, 2.9, 2.6, 2.5, 2.3, 2.2, 2, 1.9, 1.9, 1.9, 1.9],  
 range(60, 120): [4, 3.2, 2.8, 2.5, 2.4, 2.3, 2.1, 1.9, 1.8, 1.8, 1.8, 1.8, 1.8, 1.8, 1.8, 1.8],  
 range(120, 2 \*\* 100): [3.8, 3, 2.6, 2.4, 2.2, 2.1, 2, 2, 1.9, 1.9, 1.9, 1.8, 1.8]  
 }  
 **if** f\_p > get\_value(f\_t, f\_3)[f\_4]:  
 print(**f"Тест тривав {**(datetime.now() - start\_time).total\_seconds()**} сек."**)  
 n = 8 **if** n == 4 **else** 14  
 print(  
 **f"fp = {**f\_p**} > ft = {**get\_value(f\_t, f\_3)[f\_4]**}.\n"  
 f"Математична модель неадекватна з цими експериментальними даними\n"  
 f"-----------------------------------------------------------------"  
 f"\nПочнемо знову з m = {**m**} and n = {**n**}"**)  
 **return** main(m=m, n=n)  
 **else**:  
 print(**f"Тест тривав {**(datetime.now() - start\_time).total\_seconds()**} сек."**)  
 print(  
 **f"fP = {**f\_p**} < fT = {**get\_value(f\_t, f\_3)[f\_4]**}.\n"  
 f"Математична модель адекватна\n"**)  
  
main(m=2, n=14)

